Worlds, Models, and Descriptions

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Abstract. Since the pioneering work by Kripke and Montague, the term *possible world* has appeared in most theories of formal semantics for modal logics, natural languages, and knowledge-based systems. Yet that term obscures many questions about the relationships between the real world, various models of the world, and descriptions of those models in either formal languages or natural languages. Each step in that progression is an abstraction from the overwhelming complexity of the world. At the end, nothing is left but a colorful metaphor for an undefined element of a set **W** called worlds, which are related by an undefined and undefinable primitive relation **R** called accessibility. For some purposes, the resulting abstraction has proved to be useful, but as a general theory of meaning, the abstraction omits too many significant features. So much information has been lost at each step that many philosophers, linguists, and psychologists have dismissed model-theoretic semantics as irrelevant to the study of meaning. This article examines the steps in the process of extracting the pair (**W**,**R**) from the world and the way people talk about the world. It shows that the Kripke worlds can be reinterpreted as part of a Peircean semiotic theory, which can also include contributions from many other studies in cognitive science. Among them are Dunn's semantics based on laws and facts, the lexical semantics preferred by many linguists, psychological models of how the world is perceived, and philosophies of science that relate theories to the world. A full integration of all those sources is far beyond the scope of this article, but an outline of the approach suggests that Peirce's vision is capable of relating and reconciling the competing sources.

Published in *Studia Logica*, Special Issue *Ways of Worlds II*, **84:2**, 2006, pp. 323-360.

1. Models for Modality

As a *de facto* standard for modal logic, Kripke semantics is usually adopted for any kind of intensional logic, and logicians rarely ask fundamental questions about its suitability: Although a Tarski-style model is a set-theoretic construction whose elements correspond to things that are assumed to exist, what features of a Kripke-style model enable it to cross the boundary between what is and what must be, ought to be, or is believed to be? Since a Kripke model derives modality from a set of possible worlds and an accessibility relation among worlds, doesn't it presuppose a more primitive notion of modality that lies hidden in the *-ible* suffixes of *possible* and *accessible*? If not, by what magic can a Kripke model derive *ought* from *is*? If so, what makes possible worlds possible or accessible? Could that more primitive notion of modality be formalized? Can the modal axioms be derived from a more primitive notion that avoids the possible worlds and the accessibility relation? What formal mechanisms could support such a derivation? Would it be as powerful and flexible as Kripke semantics? Or more so? Could it explain the bewildering variety of axioms for modality? Could it enable different modes of modality to be expressed in the same sentence? Could it support multimodal reasoning? How?

An alternate semantics proposed by Hintikka (1961, 1963) and extended by Dunn (1973) may answer or at least clarify these questions. Instead of possible worlds, Hintikka assumed maximally consistent sets of propositions called *model sets* with an *alternativity relation* between model sets that is equivalent to Kripke's accessibility relation between worlds. Informally, Hintikka's model sets could be considered descriptions of Kripke's worlds. Formally, model sets have more structure that permits a wider range of operations. Dunn took advantage of that structure to distinguish a subset of each model set called *laws*:

1. Each model set $\mathbf{M}_i$ is considered the *facts* that describe the corresponding world w_i.
2. For each world w_i, Dunn assumed a subset $\mathbf{L}_i$ of $\mathbf{M}_i$ called the *laws* of w_i.
3. Then Dunn defined accessibility from the world w_i to the world w_j to mean that the laws $\mathbf{L}_i$ of the first world are a subset of the facts $\mathbf{M}_j$ of the second world:

   $$\mathbf{R}(w_i,w_j) \equiv \mathbf{L}_i \subset \mathbf{M}_j.$$

   According to this definition, the laws of the first world w_i remain true in the second world w_j, but they may be demoted from the status of laws to merely contingent facts.
4. Finally, Dunn showed that this construction is a conservative refinement of Kripke's theory, since any Kripke model structure ($\mathbf{W}$,$\mathbf{R}$,Φ) can be converted to one of Dunn's model structures by replacing every world w in $\mathbf{W}$ with the set $\mathbf{M}$ of propositions that are true in w and the set $\mathbf{L}$ of propositions that are necessarily true in w:

   $$\mathbf{M} = \{p \mid \Phi(p,w) = \mathbf{true}\}.$$

   $$\mathbf{L} = \{p \mid (\forall u \in \mathbf{W})(\mathbf{R}(w,u) \supset \Phi(p,u) = \mathbf{true}\}.$$

This construction generates a model in Dunn's sense for every model in Kripke's sense. It holds for the *normal models* (Kripke 1963), in which every law is also a fact, and for the *non-normal models* (Kripke 1965), in which some laws might not be facts. In deontic logic, for example, the laws are obligatory, but sinners might violate them. For such models, **R** is not reflexive because a non-normal world is not accessible from itself. (This, by the way, is one more example of the nonintuitive nature of Kripke models: it seems odd to say that any world could be inaccessible from itself, especially the real world because it happens to have sinners.)

With Dunn's construction, the accessibility relation is no longer primitive, and modal semantics does not depend on imaginary worlds. Instead, modality depends on the choice of laws, which could be laws of nature or merely human rules and regulations. Every theorem proved with Kripke's models is also true of Dunn's models, but the additional structure available with model sets enables new operations, theories, and applications to be explored. Furthermore, vague questions about the meaning of modality can be stated more precisely as "Who or what determines the laws?"

Any mention of a lawgiver threatens to invoke the twin demons of subjectivism and psychologism, which Kripke's sets seem to avoid. Montague (1967) explicitly stated his goal of replacing messy human attitudes with sets: "It has for fifteen years been possible for at least one philosopher (myself) to maintain that philosophy, at this stage in history, has as its proper theoretical framework set theory with individuals and the possible addition of empirical predicates." With sets as the ultimate ontology, everything is reducible to constructions built from a single primitive, the empty set: { }. The next step is to construct the set whose only member is the the empty set, {{ }}, or iterations of such emptiness *ad infinitum*: {{ }, {{ }}, {{{ }}}, ...}.

Like Montague, Quine preferred to reduce everything to sets, but one of his former students, who later spent many years working as an assistant to Gödel, came to deplore the emptiness of such constructions. Wang (1986) called Quine's critical, but reductionist approach *logical negativism*:

> Quine merrily reduces mind to body, physical objects to (some of) the place-times, place-times to sets of sets of numbers, and numbers to sets. Hence, we arrive at a purified ontology which consists of sets only.... I believe I am not alone in feeling uncomfortable

> about these reductions. What common and garden consequences can we draw from such grand reductions? What hitherto concealed information do we get from them? Rather than being overwhelmed by the result, one is inclined to question the significance of the enterprise itself. (p. 146)

In support of this view, Wang quoted a personal letter from C. I. Lewis about the state of philosophy in 1960:

> It is so easy... to get impressive 'results' by replacing the vaguer concepts which convey real meaning by virtue of common usage by pseudo precise concepts which are manipulable by 'exact' methods — the trouble being that nobody any longer knows whether anything actual or of practical import is being discussed. (p. 116)

As this discussion indicates, many questions about the meaning of modality are not adequately answered by Kripke semantics. In fact, the possible worlds themselves raise serious ontological and epistemological questions. Their contribution to the study of meaning is a mathematical structure whose relevance to language, communication, and the intentions of the people who communicate is certainly not obvious. The five sections of this article explore these issues and their relationship to possible worlds and to the laws and facts that describe them:

1. This first section has presented the one-to-one mapping between possible worlds and sets of laws and facts. That mapping preserves all the axioms and theorems of Kripke semantics, and it makes the laws that determine modality accessible to critical analysis.
2. The next section discusses philosophical issues about the derivation of laws and facts from the analysis and observation of worlds or situations, either real or imaginary. The conclusion is that observation alone is insufficient to derive the laws or principles that determine modal and intentional properties.
3. A more promising source of laws is Peirce's theory of signs, which covers the full range of semiotic issues from epistemology and philosophy of science to formal ontology with its applications to linguistics and computer science and finally to methods of reasoning, communication, and purposive action in and on the world.
4. Since Dunn's semantics shifts the focus of modal semantics from possible worlds to propositions, Section 4 replaces the definition of proposition as a set of possible worlds with a more fine-grained definition as an equivalence class of sentences.
5. The concluding section discusses new representations in linguistics and new methods of reasoning in artificial intelligence that become available when the formalism is expressed in terms of laws and facts.

An earlier article (Sowa 2003) presented *nested graph models* as a family of general structures for Dunn's semantics that could be specialized to versions of Kripke models, Montague models, Barcan models, and models for various temporal, modal, and intentional logics. This article explores the technical advantages and the methodological clarifications that become possible with that approach.

2. Attempts to Derive Modal Notions

Tarski's semantics derives the truth or falsity of statements in first-order logic from models consisting of a set of objects that are assumed to exist and relations that are true of those objects. But the impossibility of deriving ethical or aesthetic judgments from observations of existing objects and relations has been recognized for centuries. Yet philosophers have continued to search for methods of deriving closely related modal notions, propositional attitudes, and social relationships from abstract sets or observable data. Despite his fondness for sets as a basis for ontology, Quine (1972) recognized that Kripke's sets failed to capture the intuitive meaning of modality:

> The notion of possible world did indeed contribute to the semantics of modal logic, and it behooves us to recognize the nature of its contribution: it led to Kripke's precocious and significant theory of models of modal logic. Models afford consistency proofs; also they have heuristic value; but they do not constitute explication. Models, however clear they be in themselves, may leave us at a loss for the primary, intended interpretation.

Finding a model for a set of axioms can, by itself, prove only that the axioms are consistent relative to the axioms of the system used in constructing the model. Whatever modality is embodied in a Kripke model structure (**W**,**R**,Φ) is ultimately derived from some choice of a particular set **W** and relation **R**. The effect of the formal machinery is to expose the modality that lurks behind the chosen **W** and **R**; it does not explain the reason for the choice.

The metaphor of calling an element of **W** a *world* conjures up images of the Starship Enterprise visiting worlds with physical laws similar to our own, but with very different flora, fauna, and cultures. But Kripke's original worlds were nothing but unstructured points, and all the modal information lay buried in the primitive and unexplained accessibility relation **R**. Montague added more detail by assuming that a noun like *cow* would correspond to a function that would map any world to the set of its cows. By using the syntax of a sentence as a blueprint for composing such functions, Montague showed how a function could be constructed for the sentence that would map possible worlds to truth values. Montague's brilliant mathematical innovations distracted attention from the fact that the worlds were still undefined primitives, and there were no guidelines for defining the functions assumed for each word in the lexicon or actually applying them to any world. Barbara Partee, one of the pioneers of formal semantics, made the following observation about the hidden assumptions:

> In Montague's formal semantics the simple predicates of the language of intensional logic (IL), like *love*, *like*, *kiss*, *see*, etc., are regarded as symbols (similar to the "labels" of [predicate calculus]) which could have many possible interpretations in many different models, their "real meanings" being regarded as their interpretations in the "intended model". Formal semantics does not pretend to give a complete characterization of this "intended model", neither in terms of the model structure representing the "worlds" nor in terms of the assignments of interpretations to the lexical constants. The present formalizations of model-theoretic semantics are undoubtedly still rather primitive compared to what is needed to capture many important semantic properties of natural languages, including, for example, spatial and other perceptual representations which play an important role in many aspects of linguistic structure. The logical structure of language is a real and important part of natural language and we have fairly well-developed tools for describing it. There are other approaches to semantics that are concerned with other aspects of natural language, perhaps even cognitively "deeper" in some sense, but which we presently lack the tools to adequately formalize. (2005, Lecture 4)

As in Quine's observation about Kripke's models, Montague's models demonstrate consistency, but "leave us at a loss for the primary, intended interpretation." A lexicographer, however, would expect to find the intended interpretation in a dictionary entry.

To avoid the worlds of Kripke and Montague, Barwise and Perry (1983) proposed *situation semantics* as a theory that relates meaning to much smaller situations. They defined a proposition as a pair {σ;T} consisting of a *situation* σ and a *situation type* T, which expresses a pattern of relationships that is true of σ. According to their original definition, a situation is a region of space-time, but they found it difficult to specify the criteria for determining what region is relevant to any particular case. One of their examples was the situation of a college lecture, which illustrates the difficulty:

1. A lecture could be considered a situation bounded by a fifty-minute time period in a spatial region enclosed by the walls of a classroom. But if the time were shifted by thirty minutes, the region would include the ending of one lecture and the beginning of another. That time shift would create an unnatural "situation."
2. If the space were shifted by half the width of a classroom, it would include part of one class listening to one professor, part of another class listening to another professor speaking on a different topic, and a wall between the two lectures. That would create an even more unnatural situation than the time shift.
3. A third option would fix the coordinate system relative to the sun instead of the earth. Then the region that included the class at the beginning of the lecture would stay behind as the earth moved. Within a few minutes, it would be in deep space, containing nothing but an occasional hydrogen atom.

Even more complex situations would be needed for the referents of Greek mythology or the U.S. legal history. The first is fictional, and the second is intertwined with all the major events that happened in the United States from 1776 to the present. The space-time region for a fictional situation does not exist, and the space-time region for the U.S. legal history cannot be separated from the region of its political, economic, or cultural history.

In discussing the development of situation theory, Devlin (1991) observed that the definitions were stretched to the point where situations "include, but are not equal to any of simply connected regions of space-time, highly disconnected space-time regions, contexts of utterance (whatever that turns out to mean in precise terms), collections of background conditions for a constraint, and so on." After further discussion, Devlin admitted that they cannot be defined: "Situations are just that: situations. They are abstract objects introduced so that we can handle issues of context, background, and so on." In short, situations determine meaning, but there are no criteria for distinguishing a meaningful situation from an arbitrary chunk of space-time. The purpose of situation semantics is to derive the meaning of propositional attitudes from situations, but the choice of situation depends critically on the meaning that is supposed to be derived.

The difficulty of deriving meaning from observable data was illustrated by the shortcomings of one of the most ambitious attempts: *Der logische Aufbau der Welt* by Carnap (1928). As he explained in the preface to the second edition (1961), "The main problem concerns the possibility of the rational reconstruction of the concepts of all fields of knowledge on the basis of concepts that refer to the immediately given." To demonstrate possibility, the book begins with an analysis of *elementary experiences* (Elementarerlebnisse) and "outlines" the methods for defining concepts at three levels:

1. **Lower levels, autopsychological objects:** Detailed logical formulas for representing similarity, quality classes, part identity, part similarity, similarity between qualities, sensations, the divisions of an elementary experience, the visual field, colors, and the preliminary time order.
2. **Intermediate levels, physical objects:** No formalism, but an extended discussion "merely in a loose paraphrase" (§123) of the geometric principles involved in the derivation of "My Body" and its relationships to physical objects in a Euclidean space.
3. **Upper levels, heteropsychological objects:** A vague discussion of "other persons" with

frequent apologies for the vagueness and some hopes that behaviorism or neural science might someday fill the gaps.

For Carnap, psychological "objects" were physical objects that have a spatiotemporal location within an individual "human object" (§18). He "clarified" the notion of "intention relation" by claiming it is "nothing but" a relation between a psychological object and some other object (§164). He recognized the importance of the "sign relation", but admitted that "The construction of this relation is more difficult than any of the other relations which we have hitherto undertaken" (§141). In the 1961 preface, Carnap devoted several pages to discussing his later ideas about the lower and intermediate levels, but he said nothing about the vague upper levels, which include intentions, signs, language, communication, and social relations.

Among the more recent attempts to reduce meaning and intentionality to observable terms, Smith (1995, 1998, 1999) tried to define all aspects of human life in terms of mereological sums of physical objects and processes. His resulting ontology has two basic categories, *continuants* (physical objects) and *occurrents* (processes), which he combines by the *partOf* relation to form complex entities called *physical-behavioral units* and *social objects*:

- Examples of physical-behavioral units include "Wendy's Friday afternoon class, Jim's meeting with his teacher, your Thursday lunch, Frank's early morning swim." These units are similar to Barwise and Perry's situations, but the hyphenated adjective makes them sound more observable and therefore more objective.
- Examples of social objects include legal entities such as "juries, courts, contracts, lawsuits", cultural entities such as "works of music and literature", and human social groups such as "families and tribes, nations and empires, but also orchestras and chess clubs, battalions and football teams, as well as those more or less short-lived social groupings, which arise when strangers are formally introduced, or pair up on the dance floor." In trying to make social objects purely physical, Smith defined them as far-flung conglomerations of objects and processes. A contract, for example, would include not only the signed piece of paper, but also the people who signed it, the act of signing, and all the objects and processes involved in fulfilling the contract throughout its duration. What is missing from Smith's definition are the only things that could make it meaningful: the intentions of the people who sign the contract and carry out its provisions.

Although Smith claimed objectivity for his definitions, they violate the requirements for an effective operational test. His social objects, for example, include so many physical entities scattered over long intervals of time that itemizing them on paper is difficult and observing them in action is impossible. When arguing about a contract in a court of law, lawyers do not consult the mereological sum of physical actions, but sentences written on paper or spoken by witnesses. In Smith's examples, the fundamental requirement that determines the nature and extent of the entities involved is some sign or signs spoken by humans (e.g., "Meet in my office at 2 pm.") or recorded on paper (e.g., a deed, receipt, or contract). In every case, signs are fundamental not only to the definition of the social objects, but to their actual existence.

These examples illustrate the difficulty, if not impossibility of deriving modality or propositional attitudes from any set or structure — concrete, abstract, or imaginary — unless it had been selected or constructed on the basis of some prior intentionality. This point is independent of any position for or against *modal realism*, as formulated by David Lewis (1986): "The thesis that the world we are part of is but one of a plurality of worlds" (p. vii). Although he believed in the reality of possible worlds, Lewis admitted that there is no way to gain any knowledge of them by observation or communication.

Instead, he maintained that knowledge of possible worlds is obtained in the same way as mathematical knowledge: "we come by our opinions largely by reasoning from general principles that we already accept; sometimes in a precise and rigorous way, sometimes in a more informal fashion, as when we reject arbitrary-seeming limits on the plenitude of the mathematical universe" (p. 113). In addition to those general principles, Lewis proposed "a principle of recombination" for assembling elements of the actual world with imaginary variations. In other words, any possible worlds that are knowable or imaginable are the product of human reasoning, imagination, and selection of principles. Therefore, any modality or intentionality derived by any formalism based on them must be a transformation of whatever intentions motivated their selection or construction.

3. Semiotic Foundations

Tarski semantics for first-order logic depends on a dyadic correspondence between some pattern of symbols stated in a formal language and some pattern of objects and relations that exist in a model. Such correspondences can answer several important kinds of questions about an object, an event, or a situation: What is it? What are its properties? What is it made of? And how is it related to other things? But there's one kind of question that no amount of observation can answer with certainty: those that begin with the word *why*. For modal logic, no observations can explain why some pattern in the world might be necessary, possible, or impossible. For the behavior of humans and other animals, no observation can explain why they happen to perform one action rather than another. A thoughtful observer might be able to guess the reasons for their actions based on an analogy with his or her own experience, but the intentions themselves are not directly observable. Since humans can talk, the best way to determine their intentions is not to observe and classify worlds or situations, but just to ask them. But to take that option is an admission that Carnap's attempt to reduce mental phenomena to observable data about behavior has failed.

Among the philosophers who believe that Carnap's approach was a dead end, Searle (1983) claimed that the semantics of natural language, at least for language about anything dealing with intentionality, depends critically on the nature of the mind:

> The capacity of speech acts to represent objects and states of affairs in the world is an extension of the more biologically fundamental capacities of the mind (or brain) to relate the organism to the world by way of such mental states as belief and desire, and especially through action and perception. Since speech acts are a type of human action, and since the capacity of speech to represent objects and states of affairs is part of a more general capacity of the mind to relate the organism to the world, any complete account of speech and language requires an account of how the mind/brain relates the organism to reality. (p. vii)

In the concluding chapter, Searle claimed "there really are such things as intrinsic mental phenomena which cannot be reduced to something else or be eliminated by some kind of re-definition. There really are pains, tickles, and itches, beliefs, fears, hopes, desires, perceptual experiences, experiences of acting, thoughts, feelings, and all the rest" (p. 262). In the middle of the book, Searle formalized some of the discussion in formulas of the following kind, where p represents a proposition that describes some action or state (p. 32):

$$\text{Sorry}(p) \;\rightarrow\; \text{Bel}(p) \;\&\; \text{Des}(\sim p).$$

This formula says that if some person x is sorry for the state of affairs described by p, then x believes p

and x desires that p not be true. Since Searle claimed that mental attitudes such as sorrow, belief, and desire actually exist, each of his monadic predicates could be expanded to a triadic relation named Experience, which explicitly relates the agent to the attitude and the proposition:

$$\text{Sorry}(p) \rightarrow (\exists x{:}\text{Person})(\exists y{:}\text{Sorrow})\text{Experience}(x,y,p).$$

$$\text{Bel}(p) \rightarrow (\exists x{:}\text{Person})(\exists y{:}\text{Belief})\text{Experience}(x,y,p).$$

$$\text{Des}(p) \rightarrow (\exists x{:}\text{Person})(\exists y{:}\text{Desire})\text{Experience}(x,y,p).$$

The first formula says that the predicate Sorry about p implies the existence of a person x and an instance of sorrow y, which x experiences about p. Similarly, the second and third formulas relate the predicates Bel and Des to instances of belief and desire, which x experiences about p. In a later book, Searle (1995) was more explicit in using triadic relations to describe social relations. All his triads had the form

X *counts as* Y *in context* C.

Searle's implicit or explicit triads for describing intentions are incompatible with Smith's attempt to avoid any commitment to mental phenomena. In a public debate (Smith & Searle 2001), Smith tried to interpret Searle's constructions in terms of his own ontology, which does not admit the existence of intentions. Searle replied

> I think in the end he makes many useful points, but I also believe that he misunderstands me in certain very profound ways. I believe his misunderstandings derive from the fact that he approaches this topic with a set of concerns that are fundamentally different from mine, and in consequence, he tends to take my views as attempts to answer his questions rather than attempts to answer my questions.

Searle recognized that relations that refer to intentions have greater power and flexibility than Smith's *partOf* relation. Smith criticized that flexibility as too loose and imprecise and noted that a context itself is a social object that requires some independent definition. Both authors made valid points: Searle's book demonstrates the fundamental role of intentions in creating and sustaining social relationships, but Smith's criticism shows the need for clear distinctions and greater precision.

Philosophers since Aristotle have recognized the sign relation as triadic. The Scholastic logicians used the Latin terms *signum* for the sign, *significatio* for its significance or sense, and *suppositio* for the object it refers to. Saussure (1916) is a notable exception, who proposed a dyadic *signifier* to *signified* relation, which corresponds to the signum-significatio side of the meaning triangle, but omits Tarski's signum-suppositio or sign-object side. The dyadic versions by Saussure and Tarski have complementary weaknesses: Tarski had a clear criterion for truth, but no recognition of intention; Saussure's failure to recognize the object led theorists such as Derrida to endless levels of interpretation with no criteria for convergence.

In his study of Frege and Husserl, Mohanty (1982) noted that both of them had similar triadic definitions of signs, but with different labels. Frege's labels, *Zeichen, Sinn,* and *Bedeutung*, usually translated *sign, sense,* and *reference*, correspond to the Latin *signum, significatio,* and *suppositio*. Husserl's terms were *Bedeutung* for *sense* and *Gegenstand* (object) for *reference*. Yet Mohanty discussed their difficulties in dealing with context-dependent indexicals, such as *I, you, this,* or *that*:

> Husserl refers to indexicals and their like as threatening "to plunge all our hard-won distinctions back into confusion." These are what he calls "essentially occasional"

> expressions, in their case "it is essential to orient actual meaning to the occasion, the speaker, and the situation." (p. 57)

Although both Husserl and Frege had triadic sign relations, they recognized only one type of sign, which Peirce called a *symbol*. That triad was adequate for interpreting most content words in language, but not indexicals and other function words. They did not even consider extralinguistic signs, such as a pointing finger or smoke as a sign of fire. They faced the same dilemma as Searle: either allow vagueness by loosening the definitions of the three terms in the triad or introduce new triads to accommodate different types of signs. The latter option, however, would be *ad hoc*, unless some principled method could be found for generating new triads.

Peirce discovered a metalevel principle for generating triads during the 1860s, when he and his father, the mathematician Benjamin Peirce, worked their way through Kant's *Kritik der reinen Vernunft*. To derive his twelve categories, Kant started with four major groups — Quantity, Quality, Relation, and Modality — and divided each group into triads. As an example, Kant divided the Relation group into three categories named Inherence, Causality, and Community. While searching for a deeper principle underlying Kant's categories, Peirce noticed that Inherence could be defined by a monadic predicate that characterizes something by what it has in itself, independent of anything else; Causality would require a dyadic predicate that characterizes some reaction between two entities; and Community would depend on a triadic relation that establishes new connections among the members of the community. Following is one of Peirce's widely quoted definitions of the triad:

> First is the conception of being or existing independent of anything else. Second is the conception of being relative to, the conception of reaction with, something else. Third is the conception of mediation, whereby a first and a second are brought into relation. (CP 6.32)

Unlike Aristotle's categories, which represent types and disjoint subtypes of existing things, Peirce's are *phenomenological* categories that represent the multiple ways of perceiving, conceiving, or describing anything. They cannot be disjoint because the same thing can be described in many different ways. Furthermore, the trichotomy is not just a single triad, but a metalevel principle that can be applied repeatedly to any phenomena to generate new categories. His first application was to analyze Kant's method of subdividing the four major groups, but Peirce later applied it to any method of conceiving anything.

Many logicians failed to appreciate the force of Peirce's distinction because it seemed "obvious" to them that any triadic relation could be defined as a combination of dyads. For example, the sentence *Sue gives a child a book* could be represented with a triadic relation *gives* or with three dyadic relations that represent the *case relations* or *thematic roles* of *agent, recipient,* and *theme*:

$$(\exists x{:}\mathrm{Person})(\exists y{:}\mathrm{Child})(\exists z{:}\mathrm{Book})\\ \quad(\mathrm{nameOf}(x,\mathrm{Sue}) \land \mathrm{gives}(x,y,z)).$$

$$(\exists x{:}\mathrm{Person})(\exists y{:}\mathrm{Child})(\exists z{:}\mathrm{Book})(\exists w{:}\mathrm{Give})\\ \quad(\mathrm{nameOf}(x,\mathrm{Sue}) \land \mathrm{agnt}(w,x) \land \mathrm{rcpt}(w,y) \land \mathrm{thme}(w,z)).$$

The first formula introduces three entities of types Person, Child, and Book, states that the name of the person is Sue, and relates all three by a single triadic relation. The second formula reifies the relation by introducing the variable *w*, which refers to an instance of type Give. This method of reification, which Peirce called *hypostasis*, is often called *event semantics* and attributed to Davidson (1967). Peirce, however, not only recognized the importance of hypostasis or reification of events and attributes, he also observed that it does not change the triadic connectivity of the formula. Connectivity is not obvious when a formula is written in predicate calculus, but it is much clearer when the formula is translated to a graph. Figure 1 shows the two *conceptual graphs* that are derived from each of the two formulas in predicate calculus.

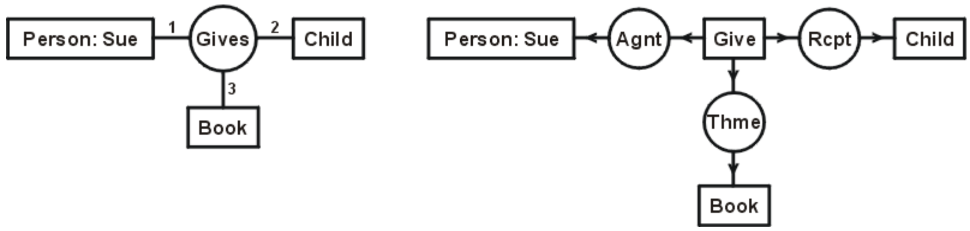

Figure 1: Two conceptual graphs for *Sue gives a child a book*

The graph on the left of Figure 1 has a single *relation node* of type Gives linked to three *concept nodes* for each of the three participants. The graph on the right replaces the relation node with a concept node of type Give and three dyadic relations of type Agnt, Rcpt, and Thme. The resulting graph has more nodes, but exactly the same connectivity: instead of three arcs attached to a relation node, it has three arcs attached to a concept node. Peirce observed that any representation of intentionality of any kind must include an implicit or explicit triadic connection, which cannot be eliminated by hypostasis. If it's not a triadic relation, it will include a triadic connection to some reified version of the intention.

Peirce viewed semiotics or as he spelled it, *semeiotic*, as the unifying theme that relates naturally occurring signs and conventional symbols to concepts and reality. Every sign is an example of Thirdness because it relates a perceptible mark (1) to some concept or other mental sign called its interpretant (2), which determines its referent or object (3). Although a sign requires an interpreter, Peirce claimed "Thought is not necessarily connected with a brain", but every thought is a sign, and every sign depends on some mind or "quasi-mind" (CP 4.551, 1906). The following definition emphasizes the independence of signs on any particular implementation:

> I define a sign as something, A, which brings something, B, its interpretant, into the same sort of correspondence with something, C, its object, as that in which it itself stands to C. In this definition I make no more reference to anything like the human mind than I do when I define a line as the place within which a particle lies during a lapse of time. (1902, p. 235)

Modern research has built on that definition to study *zoösemiotics* in animals and even *phytosemiotics* in plants (Deely 2003).

On the surface, Peirce's triadic definitions seem similar to Aristotle's, Frege's, or Husserl's. The difference, however, is that any of the three terms in the triad — the sign, the interpretant, or the object — could be further subdivided by his metalevel principle. By analyzing the method by which the interpretant relates the sign to the object, Peirce obtained the triad of *icon, index* and *symbol*: an icon represents an object by its inherent form, which resembles the intended object; an index represents its object by some physical relationship, such as a pointing finger or a dial on a meter; and a symbol represents its object by some convention, which may correspond to a concept or just a habit. The referent for an icon is a perceptible pattern; the referent for an index is a physical link; and the referent for a symbol is a convention represented by a concept, a principle, or a proposition. The indexicals that caused so much trouble for Frege and Husserl were a solved problem for Peirce; in fact, he had coined the word *indexical* for the special class of indexes that are expressed by the deictic words of natural languages.

By using the trichotomy repeatedly, Peirce derived a systematic classification of all aspects of signs and the ways they may be used. A weather vane on a steeple is, first of all, a sign of its own existence as a physical object. It serves as an index when it is used to indicate the direction of the wind. An icon of a rooster on the weather vane is a sign of the bird it resembles, but it might also be interpreted as an icon of a bird facing into the wind because its streamlined shape reduces the resistance from that direction. A rooster is also used as a symbol of early morning because its habitual crowing makes it an index of dawn. A closer inspection of the weather vane could reveal many other signs: a trademark as a symbol of the manufacturer; the composition of the metal as an index of the manufacturing process and even the source of the ore; and the design as an index of the style or the period when it was made. Finally, a weather vane on a steeple, even when made in the 20th century, could be used as a symbol of a rural 19th-century village with its close ties to the natural forces of wind and weather.

The interpretant of a sign may be a percept, a concept, or an arbitrarily complex proposition. If the sign is a sentence, the interpretant is a proposition, whose truth is determined by testing its implications against perceptions. In the following paragraph, Peirce related a Tarski-style evaluation function to the methods of perception and to the feelings associated with perception:

> A sign is only a sign in actu by virtue of its receiving an interpretation, that is, by virtue of its determining another sign of the same object. This is as true of mental judgments as it is of external signs. To say that a proposition is true is to say that every interpretation of it is true. Two propositions are equivalent when either might have been an interpretant of the other. This equivalence, like others, is by an act of abstraction (in the sense in which forming an abstract noun is abstraction) conceived as identity. And we speak of believing in a proposition, having in mind an entire collection of equivalent propositions with their partial interpretants. Thus, two persons are said to have the same proposition in mind. The interpretant of a proposition is itself a proposition. Any necessary inference from a proposition is an interpretant of it. When we speak of truth and falsity, we refer to the possibility of the proposition being refuted; and this refutation (roughly speaking) takes place in but one way. Namely, an interpretant of the proposition would, if believed, produce the expectation of a certain description of percept on a certain occasion. The occasion arrives: the percept forced upon us is different. This constitutes the falsity of every proposition of which the disappointing prediction was the interpretant. Thus, a false proposition is a proposition of which some interpretant represents that, on an occasion which it indicates, a percept will have a certain character, while the immediate perceptual judgment on that occasion is that the percept has not that character. A true proposition is a proposition belief in which would never lead to such disappointment so long as the proposition is not understood otherwise than it was intended. (CP 5.569)

Note the words *expectation*, *disappointment*, and *intended*. In various writings, Peirce observed that perception is always accompanied by some feelings — positive, negative, or neutral — toward the object of perception. Deely (2003) noted that Peirce's usage anticipates the psychoanalytic notion of *cathexis*, which binds a cognition with an emotion or affect in a unitary experience. Value judgments can be derived from perception, but from the emotional side of the cathexis, not its cognitive side.

Besides using the trichotomies to classify signs, Peirce also used them to classify mathematics, philosophy, science, and the study of signs themselves. He subdivided the field of semiotics into three subfields according to which aspect of signs each addresses:

1. **Universal grammar** is first because it studies the structure of signs independent of their use. The syntax of a sentence, for example, can be analyzed without considering its meaning, reference, truth, or purpose within a larger context. In its full generality, universal grammar defines the types of signs and patterns of signs at every level of complexity in every sensory modality.

2. **Critical logic**, which Peirce defined as "the formal science of the conditions of the truth of representations" (CP 2.229), is second because truth depends on a dyadic correspondence between a representation and its object. In further analyzing the branches of logic, he observed that induction exemplifies Secondness because it depends on a dyadic relation between propositions and reality. Deduction exemplifies Thirdness because it is determined by mediating laws that relate premises to conclusions. In looking for the missing Firstness, he discovered the principle of abduction, which generates new hypotheses, which are further tested by the methods of deduction and induction.
3. **Methodeutic** or **philosophical rhetoric** is third because it studies the principles that relate signs to each other and to the world: "Its task is to ascertain the laws by which in every scientific intelligence one sign gives birth to another, and especially one thought brings forth another" (CP 2.229). By "scientific intelligence," Peirce meant any intellect capable of learning from experience, among which he included dogs and parrots. Pietarinen (2004) pointed out that Peirce had anticipated much of the modern work on speech acts, relevance, and conversational implicatures; although he hadn't listed the principles as conveniently as Grice (1975), he discussed and analyzed versions of them in many of his writings.

Charles Morris (1938) popularized this triad under the headings *syntax, semantics,* and *pragmatics*. Unfortunately, Morris's choice of words has led to confusion, especially over the word *semantics*, which logicians use in much same sense as Peirce's term *critical logic*, but linguists use in a much broader sense. As an alternative, linguists have coined the term *lexical semantics*, which includes aspects of all three branches of semiotics.

When applied to the modes of existence, Peirce's trichotomy generates three fundamental ontological categories, which correspond to Kant's triad under Modality: Possibility, Actuality, and Necessity. As Peirce said (1908, EP 2.478), "I recognize three Universes, which are distinguished by three Modalities of Being":

1. **Ideas or possibilia**. This universe, which has a strong resemblance to Plato's forms or Whitehead's eternal objects, "embraces whatever has its being in itself alone, except that whatever is in this Universe must be present to one consciousness or be capable of being so present in its entire Being. It follows that a member of this universe need not be subject to any law, not even to the principle of contradiction." The ideas include all the possible worlds that David Lewis imagined and much more, since they need not be organized in worlds and they need not be actualizable. They even include Meinong's round squares and other ideas that are so vague, so contradictory, or so immense that they could never be actualized. All mathematical objects belong to this universe, including Cantor's infinities and even multitudes beyond anything that Cantor imagined. Any idea that any intelligent being of any species could ever conceive is in this universe.
2. **Existents**. This universe includes "Objects whose Being consists in their Brute reactions, and of, second, the Facts (reactions, events, qualities, etc.) concerning those Objects.... Every member of this Universe is either a Single Object subject, alike to the Principles of Contradiction and to that of Excluded Middle, or it is expressible by a proposition having such a singular subject."
3. **Necessitants**. "The third Universe consists of the co-being of whatever is in its Nature *necessitant*, that is, is a Habit, a law, or something expressible in a universal proposition. Especially, *continua* are of this nature.... It includes whatever we can know by logically valid reasoning." All mathematical theorems are in this universe, but the objects they characterize are either in the second universe (Existents) if they are actualized or in the first (Ideas) if they

are not. In Lewis's terms, the necessitants are the general principles and the principles of recombination for creating, organizing, interpreting, and understanding worlds, either actual or merely possible.

These three categories clarify the issues discussed by the physicist Eugene Wigner (1960) in his classic paper "On the unreasonable effectiveness of mathematics in the natural sciences." Wigner marveled at the success of mathematics in describing the universe and predicting the outcome of experiments before they had been carried out. The reason for that success can be expressed in Peirce's terms: everything existent can be described by mathematics because the infinities in the universe of ideas include more than enough options to describe the finite universe; mathematical theories in the universe of necessitants can predict the future because the infinity of all possible theories includes the formulations of every conceivable law of nature. Science is the project that searches the infinity of theories to find those that best characterize what exists and how it operates. What is truly marvelous is not the ability of mathematics to express accurate theories about the universe, but the ability of the human mind to discover some of those theories or at least useful approximations to them.

Peirce's representations of modality evolved with his development of logic. In the same paper where he presented his algebraic notation for predicate calculus, Peirce (1885) explained the semantics of counterfactual or hypothetical propositions by quantification over "states of things":

> Now, the peculiarity of the hypothetical proposition is that it goes out beyond the actual state of things and declares what *would* happen were things other than they are or may be. The utility of this is that it puts us in possession of a rule, say that "if *A* is true, *B* is true," such that should we hereafter learn something of which we are now ignorant, namely that *A* is true, then, by virtue of this rule, we shall find that we know something else, namely, that *B* is true. There can be no doubt that the Possible, in its primary meaning, is that which may be true for aught we know, that whose falsity we do not know. The purpose is subserved, then, if, throughout the whole range of possibility, in every state of things in which *A* is true, *B* is true too.

In various writings, Peirce represented necessity by a universal quantifier Π_ω and possibility by an existential quantifier Σ_ω that range over states of things indexed by the variable ω. In some writings, he read "under all circumstances" for the quantifier Π_ω.

In analyzing the development of Peirce's views of modality, Lane (forthcoming) observed that a major shift occurred in 1896, when Peirce moved from a position of weak modal realism to strong modal realism. Before then, he appeared to be satisfied with the interpretation of modality as quantification over "states of things" or "states of information." Afterwards, he considered that view "too nominalistic" and "superficial" because it did not recognize laws as active general principles that are "really operative in nature" (CP 5.100, EP 2.183). In his later writings, Peirce occasionally used quantification over states of information because it is "extremely helpful up to a certain point", but he insisted that the ultimate source of modality is the laws or general principles that determine what states are necessary or possible.

Lane's study suggests that Peirce would have preferred Dunn's semantics of laws and facts to Kripke's semantics of possible worlds, which is basically a refinement of Peirce's method of quantifying over states. Dunn's models focus on the laws that are the ultimate source of modality. They are, in fact, the necessitants of the third universe. In his later work on logic, Peirce developed *existential graphs*, which are just as expressive as his algebraic notation for predicate calculus (Peirce 1885, 1909). In adding modality to the graphs, Peirce experimented with quantification over states of information, but he preferred to use colored areas, which indicate "the nature of the universe or the universes of discourse (for several may be referred to in a single assertion)" (quoted by Roberts 1973, p. 102). With the option of allowing multiple universes in the same assertion, Peirce began to explore multimodal reasoning. Those explorations were fragmentary and incomplete, but modern systems of multimodal reasoning have not made significantly more progress.

As this survey indicates, semiotics is a rich subject that has a great deal of power for analyzing interrelationships among signs, the objects they refer to, and the people who use them for interpreting the world, acting upon it, and communicating with other people. If possible worlds have any reality, as David Lewis claimed, it could best be interpreted as the reality of sign types, such as Peirce's universe of ideas or possibilia. The facts of the actual world would be in the universe of existents, and the facts of other worlds would be in the universe of ideas. All the variations of laws that govern all the worlds, actual or not, would be in the universe of necessitants. These three kinds of elements furnish the raw material for defining meanings and relating them to the actual world. To illustrate how other theories of meaning could be interpreted as special cases of Peirce's semiotics, consider the definition of proposition in situation semantics: a pair {σ;T} consisting of a situation σ and a sign type T, which Barwise and Perry called a situation type. If T accurately describes σ and σ happens to be in the universe of existents, the proposition is true in the ordinary sense. But the proposition could also be true in some hypothetical, future, or intentional sense if σ is a situation of the proper sort in the universe of ideas. The laws, rules, principles, and habits in the universe of necessitants determine how the objects, processes, and intelligent beings in the other two universes behave and interact. This approach will be developed further in Section 5.

4. Propositions as Equivalence Classes

Any definition of modality in terms of propositions requires a precise definition of *proposition*. Informally, a proposition should represent the language-independent meaning of a sentence, but "language independence" and "meaning" are two notoriously difficult notions to formalize. Stalnaker (1976) proposed that the proposition expressed by a sentence be defined as the set of possible worlds in which the sentence is true. That definition, however, is too coarse grained: it causes all mathematical theorems to collapse into a single proposition. Yet Fermat's last theorem, for example, doesn't mean the same as 2+2=4, and it's much harder to prove. For a theory of modality based on Dunn's semantics, Stalnaker's definition cannot discriminate laws or facts that happen to be true in exactly the same worlds. With the axiom system S5, all worlds have the same laws, and they all degenerate to a single proposition. A suitable definition of proposition must make finer distinctions.

As Peirce said, a proposition corresponds to "an entire collection of equivalent propositions with their partial interpretants" (CP 5.569). To formalize this notion, a *proposition* may be defined as an equivalence class of sentences in some formal language **L** under some meaning-preserving translation (MPT) defined over the sentences of **L**. Meaning-preserving translations can be defined for any kind of language, but for simplicity, this article will restrict the definition to first-order logic. Formally, a *meaning-preserving translation* from the sentences of a first-order language $\mathbf{L}_1$ to the sentences of a first-order language $\mathbf{L}_2$ is any function *f* that satisfies the following four constraints:

1. **Invertible.** The MPT *f* must have an inverse *g*, which maps sentences from $\mathbf{L}_2$ back to $\mathbf{L}_1$. For any sentence *s* in $\mathbf{L}_1$, $f(s)$ is a sentence in $\mathbf{L}_2$, and $g(f(s))$ is a sentence in $\mathbf{L}_1$ but not necessarily *s* itself. To ensure that *f* is defined for all sentences in $\mathbf{L}_1$, the language $\mathbf{L}_2$ must be at least as expressive as $\mathbf{L}_1$. If $\mathbf{L}_2$ is more expressive than $\mathbf{L}_1$, then the inverse *g* might be undefined for

some sentences in $\mathbf{L}_2$. In that case, the language $\mathbf{L}_2$ would express a superset of the propositions of $\mathbf{L}_1$.

2. **Truth preserving.** Although the sentences *s* and *g*(*f*(*s*)) might not be identical, both must have exactly the same truth conditions: for any model **M** of the language $\mathbf{L}_1$, M entails *s* (i.e., $M \models s$) if and only if $\mathbf{M} \models g(f(s))$. Preserving truth is necessary for meaning preservation, but it is a weak condition that groups too many sentences in the same equivalence classes. Ideally, the test to determine whether two sentences "mean the same" should be "obvious." Formally, it should be computable by an efficient algorithm — one whose computation time is linearly or at worst polynomially proportional to the length of the sentence.
3. **Vocabulary preserving.** When *s* is translated from $\mathbf{L}_1$ to $\mathbf{L}_2$ and back to *g*(*f*(*s*)), the syntax might be rearranged, but the words or symbols that represent proper names and ontology must appear in both sentences *s* and *g*(*f*(*s*)). For example, the sentence *Every cat is a cat* should not be considered to have the "same meaning" as *Every dog is a dog* or *Every unicorn is a unicorn*, since the first is about cats, the second is about dogs, and the the third is about nonexistent things. This criterion could be relaxed to allow terms to be replaced by synonyms or definitions, but arbitrary content words or predicates must not be added or deleted by the translations. An admissible MPT might replace the word *cat* with *domestic feline*, but it should not replace the word *cat* with *dog* or *unicorn*.
4. **Structure preserving**. When *s* and *g*(*f*(*s*)) are mapped to a *canonical form* that uses only negation ~, conjunction ∧, and the existential quantifier ∃ as the logical operators, they must contain computationally equivalent patterns of negations and quantifiers: i.e., the decidability, provability, and computational complexity of both sentences must be the same. (Formally, the functions f_1 through f_4, which are defined below, specify what patterns are considered computationally equivalent.) The short justification for this approach is that conjunction is the simplest and least controversial of all the Boolean operators, while negation introduces serious philosophical and computational problems, which are inherited by other operators that require a negation in their definition. Intuitionists, for example, deny that ~~*p* should be considered provably equivalent to *p*. Computationally, ~~*p* and *p* have different effects on the binding of values to variables in Prolog, SQL, and other systems. For *relevance logic*, Anderson and Belnap (1975) disallowed the *disjunctive syllogism*, which is based on ∨ and ~, because it can introduce extraneous information into a proof.

The first two conditions, by themselves, specify an MPT whose propositions would be identical to Stalnaker's definition: each equivalence class would consist of all the sentences that happen to be true in a particular set of possible worlds. (Such an MPT, however, would not be efficiently computable because it would require a complete first-order theorem prover to verify whether two sentences express the same proposition.) By imposing stronger constraints, conditions 3 and 4 not only make the MPT more efficient, they also reduce the size of the equivalence classes and thereby preserve finer distinctions: they ensure that the content words or predicates remain identical or synonymous, they preserve the logical structure, and they prevent irrelevant content from being inserted. If *s* is the sentence *Every farmer who owns a donkey beats it*, then the sentence *g*(*f*(*s*)) might be *If a farmer x owns a donkey y, then x beats y*. Those sentences use different logical and syntactical symbols, but they have the same truth conditions, they have the same content words, and they have the same structure when expressed with only ∧, ~, and ∃. There are several reasons for choosing those three operators for the canonical form:

1. They are the three operators that Peirce chose to use for existential graphs, for reasons similar

to the following.

2. When sentences in natural languages are translated to FOL, those are the three most frequently occurring operators in the result.
3. They are also the natural choice for database query languages, such as SQL, because they have the least computational complexity.
4. Statements that use only conjunction and the existential quantifier are the simplest to verify by direct observation.
5. Negation adds some complexity, especially when multiple negations occur in the same sentence, but it is necessary to express full FOL.
6. Statements that contain any of the other common logical operators — disjunction, implication, and the universal quantifier — cannot be verified by direct observation.

In general, an MPT is defined as a function that translates sentences from one language to another, but for many applications $\mathbf{L}_1$ and $\mathbf{L}_2$ may be the same language. Any MPT f defined over a single language **L** partitions the sentences of **L** into a set of equivalence classes called *propositions*. A series of six MPTs f_0, ..., f_5 are defined below, which have the following properties:

- Each MPT f_i is its own inverse: $g_i = f_i$.
- Every equivalence class p in the partition of **L** determined by f_i contains exactly one *canonical sentence c*.
- For every sentence s in p, $f_i(s) = c$ and $g_i(f_i(s)) = c$.
- Any two sentences s_1 and s_2 of **L** are contained in the same equivalence class if and only if f_i maps both of them to the same canonical sentence c: $f_i(s_1) = c$ and $f_i(s_2) = c$.

A sentence s is said to *state* the proposition p if and only if s is contained in the equivalence class p.

For any language **L**, the simplest MPT f_0 is the identity function: for every sentence s in **L**, $f_0(s)=s$. This translation satisfies all four criteria for an MPT, since f_0 is its own inverse, it preserves truth, vocabulary, and structure, and it is very easy to compute. Its primary drawback is that it fails to group sentences into useful classes: every sentence in **L** is the canonical sentence of a proposition in which there are no other sentences.

In order to group sentences into larger equivalence classes, the MPT f_1 translates every sentence to a canonical sentence that uses only the three logical symbols $\wedge$, ~, and $\exists$. The inverse of f_1 is f_0 itself. The details for defining f_1 will vary with the syntax of every version of FOL, but to illustrate the method, let **L** be a conventional infix notation for FOL with the two quantifiers represented by the symbols $\exists$ and $\forall$, with negation represented by ~, and with conjunction, disjunction, material implication, and equivalence represented by the symbols $\wedge$, $\vee$, $\supset$, and $\equiv$. The translation of any sentence or subsentence s of **L** to its canonical form $f_1(s)$ is defined recursively:

1. If s is an *atom* (i.e., any relation symbol applied to its arguments), the result is s unchanged:

   $$f_1(s) \Rightarrow s$$

2. If s consists of a prefix operator (negation or quantifier) applied to a subsentence u, the result is

the transformed operator applied to the translation of *u*:

$$f_1(\sim u) \Rightarrow \sim f_1(u)$$

$$f_1((\exists x)u) \Rightarrow (\exists x)f_1(u)$$

$$f_1((\forall x)u) \Rightarrow \sim(\exists x)\sim f_1(u)$$

3. If *s* consists of a dyadic operator ($\wedge$, $\vee$, $\supset$, or $\equiv$) applied to two subsentences *u* and *v*, the result is a transformation of the operator applied to the translations of *u* and *v*:

$$f_1(u \wedge v) \Rightarrow f_1(u) \wedge f_1(v)$$

$$f_1(u \vee v) \Rightarrow \sim(\sim f_1(u) \wedge \sim f_1(v))$$

$$f_1(u \supset v) \Rightarrow \sim(f_1(u) \wedge \sim f_1(v))$$

$$f_1(u \equiv v) \Rightarrow (\sim(f_1(u) \wedge \sim f_1(v))) \wedge (\sim(f_1(v) \wedge \sim f_1(u)))$$

The function f_1 satisfies the criteria for an MPT: it has an inverse f_1; it preserves truth because every step of the translation expands one operator according to its definition in terms of $\wedge$, ~, and $\exists$; it leaves the content words and symbols unchanged; and it preserves the structure of a sentence when mapped to $\wedge$, ~, and $\exists$. For any sentence *s*, $f_1(s)$ is a unique canonical sentence of the equivalence class defined by f_1: every sentence in **L** that is mapped to that canonical sentence is said to state the same proposition. For all logical operators except $\equiv$, the time to convert any sentence to the canonical sentence is linear. If a sentence contains a large number of equivalences, the translation time may be exponentially proportional to the number of equivalences. Such sentences, however, are extremely rare; when people write $p \equiv q \equiv r$, they usually intend it as the conjunction $p \equiv q$ and $q \equiv r$, which merely doubles the translation time.

The function f_1 would classify sentences of the form $p \wedge q$ and $q \wedge p$ in different equivalence classes because its definition does not reorder any of the subsentences. To group such sentences in the same equivalence class, the function f_2 makes the same symbol replacements as f_1, but it also sorts conjunctions in lexical order according to some encoding, such as Unicode. As a result, arbitrary permutations of conjunctions map to the same canonical sentence and are grouped in the same equivalence class. Since disjunctions are defined in terms of of conjunctions, the sentences $p \vee q$ and $q \vee p$ would also be in the same class. Since a list of N terms can be sorted in time proportional to (N *log* N), the function f_2 would take just slightly longer than linear time.

Ideally, sentences that differ only in the names assigned to their bound variables, such as $(\exists x)\mathrm{P}(x)$ and $(\exists y)\mathrm{P}(y)$, should also be grouped together. Therefore, the function f_3 should rename the variables to a fixed sequence, such as x_1, x_2 , ... That renaming must be done before conjunctions are sorted; the simplest method would be to move all quantifiers to the front by mapping every sentence to *prenex normal form* and then naming the variables in the order in which they appear. The function f_3 would first convert sentences to prenex normal form and rename variables, then it would perform the translation method of f_1 and sort conjunctions according to the method of f_3. For most sentences, the renaming process is very fast, but it can slow down for sentences with large numbers of variables in complex patterns; such sentences, however, are usually rather rare.

The function f_3 is not yet an ideal MPT because it would assign sentences of the form u, $u \wedge u$, and $u \vee u$ to different equivalence classes. To group such sentences together, the function f_4 would perform exactly the same translations as f_3 followed by one additional step: deleting duplicate conjunctions. Since f_3 has already sorted conjunctions in lexical order, the deletion can be performed in linear time just by checking whether any conjunct is identical to the one before it; if so, it would be deleted from the result. This process would also delete duplicate disjunctions since the method of f_1 would have already mapped disjunctions to combinations of conjunctions and negations.

For most purposes, the function f_4 is a good MPT for grouping sentences that "say the same thing." As an example, consider the following sentence, which Leibniz called the *Praeclarum Theorema* (splendid theorem):

$$((p \supset r) \wedge (q \supset s)) \supset ((p \wedge q) \supset (r \wedge s))$$

The function f_3 would translate it to the following canonical sentence:

$$\sim((\sim(p \wedge \sim r) \wedge \sim(q \wedge \sim s)) \wedge \sim(\sim(p \wedge q) \wedge \sim(r \wedge s)))$$

This sentence is not as readable as the original, but it serves as the canonical representative of an equivalence class of f_3 that contains 864 different, but highly similar sentences. The function f_4, which deletes duplicate conjuncts, can relate infinitely many sentences to the same form. Such transformations factor out accidental differences caused by the choice of symbols or syntax.

To account for synonyms and definitions, another function f_5 could be used to replace a word or relation such as *cat* with its definition as *domestic feline*. If recursions are allowed, the replacements and expansions would be equivalent in computing power to a Turing machine; they could take exponential amounts of time or even be undecidable. Therefore, f_5 should only expand definitions without recursions, direct or indirect. Definitions of this form are common in database systems, in which a fixed set of relations are privileged, and *virtual relations* may be defined in terms of the privileged relations. With such restrictions, any sentence that uses virtual relations could always be expanded to a form that only uses relations in the privileged set. Therefore, the function f_5 would first expand all virtual relations to the privileged set before performing the translations defined for f_4. Since no recursion is involved, the expansions would take at most polynomial time.

In summary, an open-ended number of meaning-preserving translations could be defined, any of which would be suitable for supporting Dunn's semantics. For ease of computing, the availability of efficient algorithms for computing a unique canonical form for any proposition should be considered in the choice of MPT. If no definitional mechanisms are available in the given language **L**, MPT f_4 would be a good choice. If the language supports nonrecursive definitions, f_5 could be used. For languages that go beyond FOL, an extended version of f_4 or f_5 could be defined to accommodate the additional operators and structure. Much more important than choosing any particular version of MPT is the decision to define propositions as equivalence classes of sentences.

5. Making Possible Worlds Meaningful

One of the oldest controversies about Aristotle's categories was whether they represent the kinds of things that exist or the way people perceive, think, and talk about things that exist. Theophrastus, Aristotle's successor as head of the Lyceum, said that the categories were intended in all those ways — in modern terms, ontological, epistemological, and lexical. Yet the fragmented methodologies of those subjects are scattered across the fields of philosophy, linguistics, and artificial intelligence, in each of which the researchers who work on formal semantics or lexical semantics are disjoint sets. For linguistics, Partee (2005) hoped that "these different approaches can be seen as complementary and not necessarily antagonistic."

One reason for hoping that Peirce's semiotics can help relate the fragmented subfields is that the scope of his research was as broad as Aristotle's. In addition to his research on mathematics, physics, and logic, he had been an associate editor of the *Century Dictionary*, for which he wrote, revised, or edited over 16,000 definitions. The combined influence of logic and lexicography is apparent in a letter he wrote to to B. E. Smith, the editor of that dictionary:

> The task of classifying all the words of language, or what's the same thing, all the ideas that seek expression, is the most stupendous of logical tasks. Anybody but the most accomplished logician must break down in it utterly; and even for the strongest man, it is the severest possible tax on the logical equipment and faculty.

In this remark, Peirce equated the lexicon with the set of expressible ideas and declared logic the primary means of analysis. Unlike Frege, Husserl, and Russell, he did not avoid the challenge of characterizing the language people actually use by escaping to some purified realm of formal semantics or ontology.

Semiotics is a unified subject that addresses all possible uses of signs by all possible species. Various students of the subject may prefer to analyze different aspects or to adopt a philosophical, mathematical, or applied approach, but academic compartmentalization should not create artificial barriers. Following are some of the issues:

1. Dunn's semantics of laws and facts is an important part of the synthesis because it enables any formal method based on possible worlds to be mapped to an equivalent method based on laws and facts, which are easier to incorporate into a semiotic theory or a computational system. In fact, most AI systems that implement some version of possible worlds already represent them as sets of statements that describe those worlds.

2. Peirce's views on the reality of possibilia are compatible with David Lewis's views on the reality of possible worlds. Furthermore, Peirce's emphasis on the reality of generals, which are of the same nature as laws, accommodates an integrated combination of Dunn's laws with Lewis's worlds.

3. Vogt (2002) claimed that the so-called *symbol grounding* problem, as formulated by Harnad (1990) had already been solved by Peirce. By relating the sign to both its object and its interpretant, signs are grounded, directly or indirectly, in physical reality. Signs, however, presuppose some version of epistemology or philosophy of science for characterizing exactly how the triadic relation is determined in any particular instance.

4. Peirce did have a detailed theory of epistemology for everyday knowledge and for the most advanced science. For the first, he called the methods *coenoscopy* (from the Greek *koinos* [common]) and for the second *idioscopy*, which depends on sophisticated instruments whose

measurements are related via mathematical models to scientific theories. The idioscopic methods of science cut through Kant's veil of phenomena to reach the noumena or things in themselves. But Peirce's admission of *finite fallibility* is the claim that there is no way of knowing which, if any, of the idioscopic measurements characterize physical reality or just one more veil between coenoscopic perception and whatever happens to be independent of the senses, the minds, and the technology of any species.

5. The emotions that accompany any perception or action are the ultimate source of the aesthetic, ethical, and semiotic value judgments associated with the cognitive content. This binding of cognition with emotion — in modern terms, cathexis — is the foundation for any aspect of meaning that involves purpose, belief, desire, hope, fear, love, hate, or any other form of intentionality. To the extent that different individuals have similar emotional bindings to similar cognitions, the derived value judgments can have some degree of objectivity.
6. A dozen years after his pioneering work on predicate calculus (Peirce 1885), he switched to *existential graphs* as his preferred version of logic. Although the graphs are equivalent to predicate calculus in expressive power, they exhibit the relational structure and the nests of contexts more clearly. In fact, the nested contexts are isomorphic to both *conceptual graphs*, which Sowa (1984) explicitly based on Peirce's graphs, and *discourse representation structures*, which Kamp (1981) invented independently.
7. During the last decade of his life, Peirce developed his *tinctured existential graphs*, which use twelve colors for the different modes of being or "universes of discourse": four colors for modes of actuality; four for modes of possibility; and four for modes of intentionality (Roberts 1973). Unlike the modern theories that treat only one modality, Peirce allowed multiple colors to be mixed in the same graph in order to relate "multiple universes of discourse."

These seven points are a rough grouping of ideas that Peirce developed in many thousands of pages that have been published and many more thousands of pages of still unpublished manuscripts.

One controversy that semiotics clarifies are the many options for describing things and events from various perspectives. As examples, Gangemi et al. (2003) maintain that the terms *vase* and *lump of clay* have different identity criteria; therefore, they imply two distinct objects that happen to occupy the same location. Others maintain that the distribution of matter takes precedence over any method of describing it: if two descriptions characterize the same matter, they must describe the same object. Peirce, however, would say that anything can be described in any number of ways at either a coenoscopic or an idioscopic level. Any choice of words or other signs depends on the intentions of some viewer who might choose one perspective rather than another. That choice is not purely subjective, since there are objective, but species-specific criteria for preferring one to another (Deely 2003). A bee, for example, might ignore the vase and focus on the flowers in the vase, while a dog might push the flowers aside and drink the water that some human had put there for a very different purpose. Each perspective depends on the intentions of some individual of some species, and any question about the priority of one perspective over another cannot be answered without considering the intentions of the questioner.

Peirce's phenomenological trichotomy supports a methodology for classifying and relating different perspectives. For example, an object may be recognized as a dog, cat, man, or woman by directly observable properties of the individual (Firstness), but the individual cannot be recognized as a pet, stray, owner, or employee without some evidence of an external relationship (Secondness). The corresponding Thirdness is expressed by nouns such as *ownership* or *employment*, which involve intentions marked by signs, such as a contract, a handshake, or a habitual pattern of behavior by pet and owner. Verbs can be classified by the same principles as nouns: whether they signify some directly observable event (Firstness); some indirectly related effects (Secondness); or the mediating intentions (Thirdness). The next three sentences describe the same act in each of those ways:

1. Brutus *stabbed* Caesar.
2. Brutus *killed* Caesar.
3. Brutus *murdered* Caesar.

An act of stabbing can be recognized by objective criteria at the instant it happens. That is a classification by Firstness, since no other entities, events, or mental attitudes are involved. But an act of stabbing cannot be identified as killing unless a second event of dying occurs. Caesar had time to ask "Et tu, Brute?" before the stabbing could be interpreted as a killing (Secondness). Murder is Thirdness that depends on the intention of the agent. Determining whether an act of stabbing that resulted in killing should be considered a murder may depend on subtle clues, whose interpretation may require a judge, a jury, and a lengthy trial. For lexical semantics, these examples show the potential of semiotics for classifying and defining the words that describe any perspective on any world.

For formal semantics, graph logic facilitates a shift from a functional representation, as in Montague grammar, to a relational representation, as in conceptual graphs or Kamp's DRS. Formally, every function is a relation, but not every relation is a function: a functional notation, such as $y = f(x_1,...,x_n)$, can always be converted to an equivalent relational form, such as $f(x_1,...,x_n,y)$, with the added constraint that there is a unique y for every combination of $x_1,...,x_n$; but if that constraint is not true, the inverse conversion is not possible. In both linguistics and programming languages, a functional representation has the advantage of supporting *compositional semantics*: the derivation of the semantic interpretation of a sentence as a function of the interpretation of its phrases and subphrases. Unfortunately, context-dependent ambiguities can destroy functionality.

To preserve functionality, Montague (1970) removed the ambiguities from his "fragment" of English by inventing an elegant technique of *underspecification*: he left quantified noun phrases in an unresolved state between an intensional or an extensional interpretation until the verb type could determine one or the other. The many versions of *unification grammars* leave the representation underspecified by providing slots for variables whose values may be determined later. Those techniques, however, are defeated by the enormous flexibility of natural languages, which have ambiguities that may require structural changes that are far more complex than inserting values in slots. Even worse than ambiguity is ellipsis, which might omit any part of a sentence that a native speaker could infer or at least guess. Finally, the compositional methods, which are designed for languages with a fixed word order such as English, break down when applied to languages with free word order, such as classical Latin or modern Russian.

All these difficulties cause the process of interpreting language to resemble puzzle solving: the words and background information are like pieces in a jigsaw puzzle that are scrambled in an unpredictable order instead of a strictly regimented sequence. Puzzle solving requires *constraint satisfaction* methods, which are more readily accommodated by relational representations and are just as logical as functional composition. Conceptual graphs have been used as the semantic representation with both compositional and puzzle-solving methods. With any method, the semantic patterns associated with each word sense are represented by *canonical* conceptual graphs (Sowa 1976, 1992), which serve as the pieces of the semantic puzzle. The structure of the lexicon, as described by Sowa (1984), is illustrated in Figure 2.

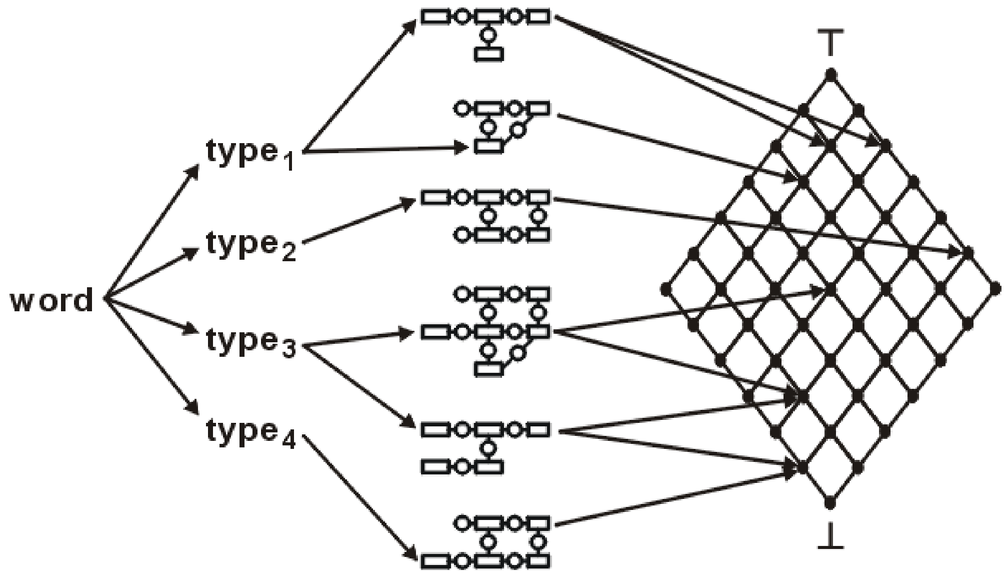

Figure 2: words → senses → canonical graphs → theories

The dotted arrows in Figure 2 link the word types of any language to an open-ended number of word senses or concept types, each of which is linked to one or more canonical graphs, which are puzzle pieces of possibly different shapes. At the right is a lattice of all possible theories, each of which represents a collection of axioms, theorems, and facts about some domain or subdomain in one of Peirce's universes: ideas, existents, or necessitants. Each canonical graph maps to one or more theories in which its logical pattern is used. Following are the four kinds of entities shown in Figure 2:

1. **Words**. Every natural language consists of meaningful units called words or *morphemes*. In some languages, every morpheme is written as a separate word, but other languages group multiple morphemes into a single word.
2. **Senses**. Each word or morpheme has one or more possible meanings, called *senses*. Most dictionaries list a few senses for each word, but Cruse (2000) maintained that any word can have an open-ended number of highly domain-dependent *microsenses*. In conceptual graphs, the senses or microsenses are represented by concept or relation *types*, which are organized in a hierarchy (partial ordering) by the operations of generalization and specialization. Some types may be expressed by word senses in several different natural languages, but others might not be expressible by a single word in any language. Way (1991) studied the use of metaphor as a means of extending the type hierarchy dynamically, and Sowa (2000) treated metaphor as one of the operators for navigating the lattice of theories, either in human thought or in computer implementations.
3. **Canonical graphs**. Each word sense or concept type has one or more canonical graphs, which represent the typical patterns in which that concept type occurs, either in natural language expressions or in the axioms of some theory. Canonical graphs represent the *selectional constraints* on the word senses or concept types that may be interconnected in any pattern, but they can never rule out new options that may result from changes in the world or innovations in language use.
4. **Theories**. The logical pattern expressed by any canonical graph may occur in zero or more theories, which characterize mathematical structures, domains of knowledge, or just fragmentary thoughts or ideas that might someday evolve into more complex hypotheses. The lattice shows how each theory is related to every other: as a generalization, specialization,

sibling, or distant cousin. New theories can be added at the top, bottom, or middle levels of the lattice at any time without affecting any reasoning that involves other theories in the lattice. The complete lattice of all possible theories is infinite, but only a finite subset could ever be conceived in any brain or be represented in any computer system.

The puzzle pieces may be expressed in a logical notation, but the labels on the nodes of a graph or the predicates of a formula cannot be limited to a fixed lexicon or a predefined formal ontology. The labels are signs, whose relations to any world, real or imaginary, must be determined by the semiotic processes. Different theories may use the same labels, but theories on different branches of the lattice may define the labels in incompatible ways. The partial ordering of theories ($T_1 \leq T_2$) could be specified by model-theoretic entailment ($T_1 \models T_2$) or by provability ($T_1 \vdash T_2$). For simplicity, the theories could be restricted to some version of first-order logic, or they might be extended to higher-order logic. In any case, no modal operators should appear in any theory, since modality can be handled by Dunn's techniques.

When Tarski (1933, 1936) wrote his famous papers on the concept of truth in formalized languages, his models were mathematical structures that entailed propositions stated in first-order logic. But when formal semanticists write $w \models p$ or $\sigma \models p$ and claim that w is a world, σ is a situation, and p is a proposition expressed in a natural language, they are ignoring Kant's struggle with the problems of phenomena and noumena and the two centuries of phenomenology, epistemology, and philosophy of science written in response to Kant. Figure 3 illustrates the issues: on the right are some axioms for a theory stated in FOL, and in the middle is a Tarski-style model consisting of a set of nodes that represent entities and a set of arcs that represent relationships among those entities. On the left is the world with all its complexities of noumena. The evaluation function Φ determines the truth value of a proposition in the theory on the right in terms of the model in the middle. That model is, at best, an approximation to some aspect of the world selected for some purpose by some human.

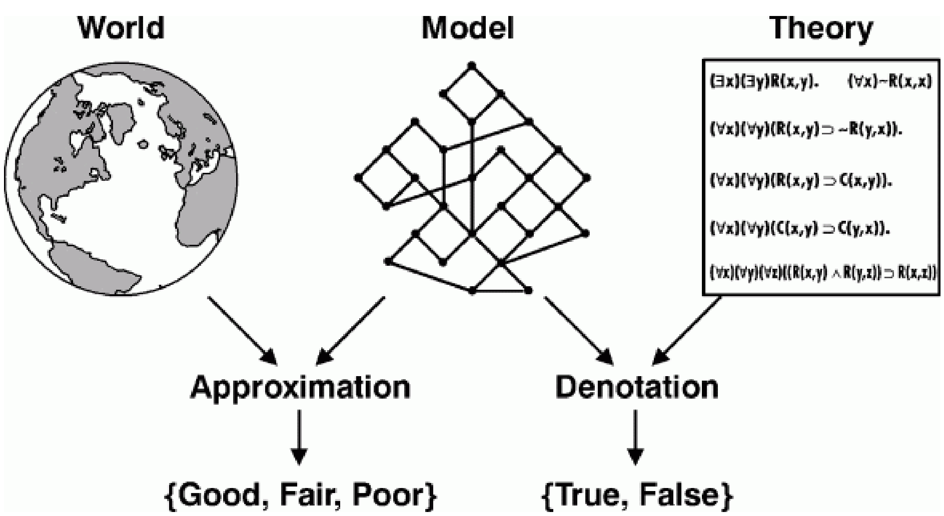

Figure 3: The world, a model, and a theory

The triad of theory, model, and world in Figure 3 is an example of a Peircean sign relation: the theory is a complex sign, the model in the middle is one of many possible interpretants of the theory, and the referent is some aspect of the world that resembles the model to some degree of approximation. That diagram suggests a way of relating Peirce's semiotics to the ontology of processes by Whitehead (1929) and the *language games* of Wittgenstein (1953):

1. The ultimate nature of the physical world is process-like, and what people call objects are projections of semiotic structures (e.g., mental models or mathematical constructions such as the digram at the center of Figure 3) onto what Whitehead called repeated occurrences of similar event types.
2. What Wittgenstein called a language game is a set of conventions, such as Dunn's laws or Peirce's necessitants, that serve as the axioms of some theory that characterizes a family of possible models of the world. They may be mental models, models represented on paper, or models consisting of data structures and programs in a computer system. The semiotic system, neural or artificial, maps those models to the vocabulary and syntax of either a natural language or some other symbolic system; e.g., a computer language; a game such as chess, bridge, or baseball; or any other conventional behavior pattern in society, e.g., traffic conventions supported by a system of roads, signal lights, intersections, lane markers, and barriers.
3. A proposition expressed in some semiotic system such as a natural language is true if its denotation in terms of the model is true and if the model corresponds to some aspect of the world that is adequate for its intended purpose. The degree of precision or vagueness of a true proposition depends on the adequacy of the correspondence between the model and the world. Tarski's logical models can never be vague, but they are always approximations. As the engineer and statistician George Box (2005) said, "All models are wrong; some models are useful."

As this article has shown, semiotics goes beyond relationships among signs to relationships of signs to the world and to the agents who observe the world, interpret it in signs, and use the signs to plan further actions upon the world. For artificial intelligence, this connection of signs to perception, action, and language supports the *symbol grounding* necessary for making the language processing by AI systems and the actions by AI robots meaningful in the same sense that the words and actions of humans are meaningful. The combination of semiotics with Dunn's semantics of laws and facts provides a theoretical foundation for modality and intentionality that captures more of the intended interpretation than a undefinable relation **R** over an undefined set **W**. An important promise of this combination is the ability to support multimodal reasoning as a kind of metalevel reasoning about the source of the laws and facts. Instead of complex axioms for each mode with even more complex interactions between modes, it enables the laws to be partitioned in a hierarchy that represents grades of necessity or levels of entrenchment: logical, physical, economic, legal, social, cultural, or personal (Sowa 2003). Exploring the full implications of Peirce's semiotics is far beyond the scope of this article, but the outline presented here suggests a wealth of resources waiting to be developed.

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